# Voltage Induced in the Rogowski Coil due to a Threading Current

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## Rogowski Model

The Rogowski coil is modeled as a toroid of inner radius $R$, and with $N$ tightly wound turns. The cross sectional area is considered to be a rectangle of dimensions $A$ and $L$. See Figure 1.

The threading current $I$ is assumed to be carried by a conductor within the toroid. The Faraday's Law of Induction is employed to find a closed form expression for EMF induced in the coil due to the time varying nature of the threading current. Recall that, according to Maxwell-Faraday Equation, the induced electric field along a path enclosing a time-varying magnetic flux density is given by

$$\oint \boldsymbol{E}.\boldsymbol{dl} = -\frac{\partial \phi_{s,t}}{\partial t} \tag{1}$$

This analysis assumes that the threading conductor is at the center of the toroid and perpendicular to its plane, but note that it is possible to solve for the general case of the conductor being at any arbitrary position and angle in the toroid. The answer is same in all cases.

## Faraday's Law

To obtain the total flux through the Rogowski due to the threading current, consider flux through an infinitesimal cross-sectional area of the coil at an angle $\theta$, as shown in Figure 2. The number of turns in that area is then

$$dT = \frac{N}{2\pi} \cdot d\theta \tag{2}$$

Within that area, the flux density can be had using Ampere's Circuital Law:

$$\oint \boldsymbol{B}.\boldsymbol{dl} = \mu_0 I_{Enc} \tag{3}$$

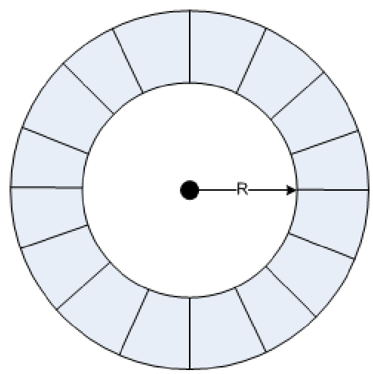


Figure 1: Rogowski Coil - Top View

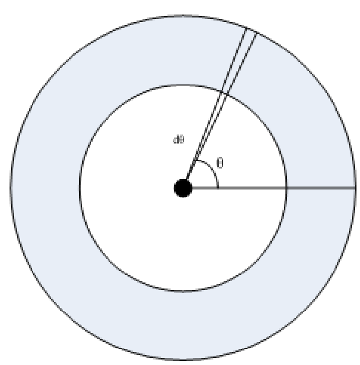


Figure 2: Infinitesimal Area at Angle $\theta$

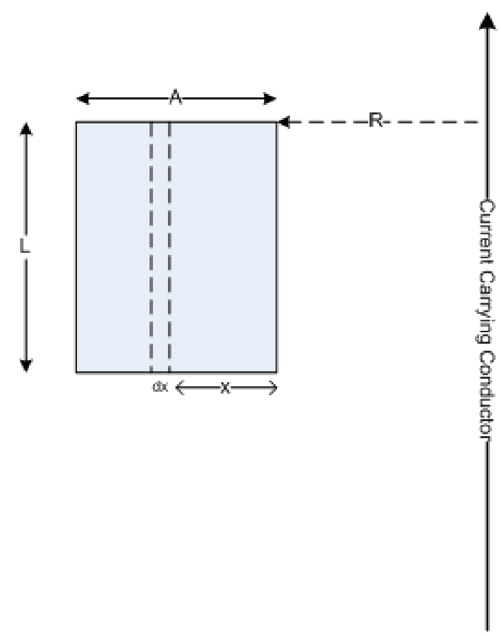


Figure 3: Rogowski Cross Sectional Area

This gives the flux through a small sliver of area $L.dx$ (see Figure 3) to be

$$\phi_{\theta,x} = \frac{\mu_0 IL}{2\pi(R+x)} \cdot dx \tag{4}$$

The total flux in the coil is obtained by integrating over the entire cross-sectional area and then by integrating over the angular range of the coil to cover all $N$ turns. The expression is

$$\phi = \oiint \phi_{\theta,x} dx \cdot dT \tag{5}$$

$$= \iint_{\theta=0,x=0}^{\theta=\pi,x=A} \frac{N}{2\pi} \cdot \frac{\mu_0 IL}{2\pi(R+x)} \cdot dx \cdot d\theta \tag{6}$$

This reduces to

$$\phi = N \cdot \frac{\mu_0 IL}{2\pi} \cdot ln\left(1 + \frac{A}{R}\right) \tag{7}$$

Ultimately, the voltage induced in the Rogowski is given by

$$EMF = -\frac{\partial \phi_{s,t}}{\partial t} \tag{8}$$

$$= -N \cdot \frac{\mu_0 L}{2\pi} \cdot ln\left(1 + \frac{A}{R}\right) \cdot \frac{dI}{dt} \tag{9}$$

Note that the transfer function of the Rogowski, $N \cdot \frac{\mu_0 L}{2\pi} \cdot ln\left(1 + \frac{A}{R}\right)$, is dependent only on the geometry.